Coherent coupling between a three-level atom and structured reservoir

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Abstract

We study the spontaneous emission (SE) spectra of a three-level Λ-type atom in photonic crystals. The air-band, dielectric-band, and free-space vacuum reservoirs coupled with the atom are considered. With the consideration of the phase difference between the air-band and dielectric-band fields, two band reservoirs become coherent and can interfere with each other. This interference effect depends on the embedded position of the atom and the band-gap width and can affect the SE spectrum. The effect of this interference on the SE spectrum is shown.

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I. INTRODUCTION

Photonic crystals (PCs) are periodic dielectric structures which can create photonic band structures in the frequencies of electromagnetic radiation. Between two photonic bands, there exist a photonic band gap (PBG) to prohibit those electromagnetic waves, whose frequencies are within the PBG, propagating inside the periodic dielectric materials. In the PBG structure, the band above (below) the PBG is called the air (dielectric) band due to the high-frequency (low-frequency) electric fields concentrating their energy in the low-dielectric (high-dielectric) regions. Due to a $\pi/2$ phase difference, the electric field of the air band changes with the electric field of the dielectric band coherently.

Because of the existence of the PBG, the dispersion of electromagnetic waves inside the PCs is strongly modified, and the density of electromagnetic-field states (DOS) shows a discontinuity. Due to this DOS discontinuity, atoms and photons inside PCs are strongly entangled. For a two-level atom embedded in PCs, many interesting phenomena have been discovered when the transition frequency of the atoms near the air band edge. There are the inhibition of spontaneous emission (SE) [1] and the coherent control of SE [1]. The SE from a three-level system has also attracted a lot of interest in the past decade. John and Quang studied the properties of a three-level $\Lambda$-type system in Ref. 1, Bay and Lambropoulos described the spontaneous emission spectrum for a three-level $\Xi$-type system in Ref. 2, and Paspalakis et al. discovered the coherent phenomena of the occurrence of dark lines in the SE spectrum for a $\Lambda$-type system in Ref. 3.

Many of earlier studies of the SE in PCs was concentrated on the one band approximation and ignored the fact that the photon-atom interaction strength inside the PCs depends on the position of an atom located [4]. One band approximation is valid when the transition frequencies of the atoms is near the air or dielectric band edge and the PBG is relatively wide. If the band gap is narrow, the photon-atom coupling has to consider both electric fields from the air-band and dielectric-band reservoirs, which is referred as "the PBG reservoirs". Existence of both reservoirs leads to a stronger photon-atom coupling. Two distinct coherent phenomena can occur in the SE, absorption, and dispersion properties of a $\Lambda$-type atom where one transition interacts near resonantly with a double-band photonic crystal [3]. The SE spectrum exhibits two dark lines. The atom can become transparent to a probe laser field coupling to the adjacent free-space transition. These two-band studies assumed that...
the atom inside PCs interacted with two independent PBG reservoirs. The photon-atom coupling strengths from both reservoirs were the same strength and independent of the atomic position. In fact, the photon-atom coupling strength depends on the electric-field intensities of two reservoirs and is dependent on the atomic position [5, 6].

Quantum interference, one of the basic features of quantum mechanics, provides many counterintuitive effects [8] and applications of quantum optics. The electromagnetically-induced transparency of Harris and co-workers [7], lasing without inversion [9, 10], nonlinear optics without phase matching [7], and suppression of atomic decay by spontaneous emission [11] are some of them. In a multilevel atomic system, quantum interference can also lead to many novel effects, for example, absorption reduction and cancellation and spontaneous emission reduction and cancellation [11–13].

In this paper, we study the SE spectra of a three-level Λ-type atom embedded in the PCs. One transition level of the atom lies inside the PBG. Although such kind of the system was studied by Angelakis et al. [3], their study was concentrated on how the SE spectra were affected by the square-root singularity of the PBG DOS near the band edge. The coherent phenomenon of electric fields from both reservoirs is ignored. On the other hand, we consider the coherence of the electric fields from the PBG reservoirs. We find that there exists quantum interference via the atom between two band fields. Such quantum interference is enhanced for the narrow PBG materials. The consideration of the coherence of the PBG reservoirs leads us naturally to show that the SE spectra of the PBG materials depend on the relative position of an embedded atom in a Wigner-Seitz cell of the photonic crystal.

The paper is organized as follows. In section II, We derive the atom-field coupling Hamiltonian, where the position dependence of the atom-field coupling constants is considered. We believe that this position dependence of the atom-field couplings is induced by the phase difference of the air-band and dielectric-band fields at different locations of the embedded atom. In section III, we calculate the SE spectrum in the free space vacuum and discuss its properties along with the different band gap widths and the relative position of the atom. We show that the coherent property of the system is enhanced as the gap becomes narrower. The spectral shape changes with the relative position of the atom. Finally, we summarize our results in section IV.
II. ATOM-FIELD COUPLING HAMILTONIAN

When the system of an atom embedded in a photonic crystal and coupling with a radiation field is considered, the quantized interaction Hamiltonian is

$$H = -\frac{e}{m} \vec{P} \cdot \vec{A}$$

where \( e \) and \( m \) are the charge and mass of the single electron in the atom, \( \vec{P} \) is the momentum of the electron without spin, the radiation field is described by the vector potential \( \vec{A} \), and the Coulomb gauge \( \nabla \cdot \vec{A} = 0 \) is considered. For the field modes from the coherent reservoirs of a photonic crystal, the interaction Hamiltonian could be expressed as [14]

$$H_{PC} \approx -\frac{e}{m} \vec{P} \cdot \vec{A}_{PC}(\vec{r}_0, t) = -i\hbar \sum_{n,k} g_{n,k} \left[ \hat{a}_{n,k} e^{-i\omega_k t} |j\rangle \langle i| + \hat{a}_{n,k}^+ e^{i\omega_k t} |i\rangle \langle j| \right]$$

where \( \vec{r}_0 \) is designated to be the position of the atomic nucleus. Note that \( g_{n,k} = \frac{\omega_{ij}}{\hbar} \left( \frac{\hbar}{2\omega_k V} \right)^{1/2} \vec{d}_{ij} \cdot \vec{E}_{n,k}^* (\vec{r}_0) \) is the position-dependent atom-field mode coupling constant, while the coupling constant with the free-space vacuum modes is independent of the position of the atom. Here \( V \) is the volume of a unit cell of the lattice, \( \vec{d}_{ij} \) is the atomic dipole moment for the transition \( |i\rangle \to |j\rangle \) with basis vectors \( \{|i\rangle\} \) forming a complete orthogonal set of the atomic system, and \( \omega_{ij} = \omega_i - \omega_j \) is the atomic transition frequency between levels \( |i\rangle \) and \( |j\rangle \). We shall assign \( \vec{E}_{a,k}^* (\vec{r}_0) \) and \( \vec{E}_{d,k}^* (\vec{r}_0) \) as the atom-coupling fields from the air band and dielectric band, respectively.

When the physical origin of the PBG is discussed, Joannopoulos et al. [5] suggested the electromagnetic variational theorem and found that the low-frequency modes concentrate their energy in the high dielectric constant region and the high-frequency modes in the low dielectric region. The PBG arises due to this difference in field energy location. That is, there exists a phase difference between the mode fields from two band reservoirs. To simulate this phase difference of the fields from the different band reservoirs, we assume that there exists only one eigen-mode field \( \vec{E}_k \) for two band reservoirs. The distributions of field intensities of two band reservoirs are given by

$$\vec{E}_{a,k}^* = \vec{E}_k^* \cos \theta, \vec{E}_{d,k}^* = \vec{E}_k^* \sin \theta$$
where the angle \( \theta \) is determined by the position of the atom \( \vec{r}_0 \), i.e., \( \theta = \theta(\vec{r}_0) \). We then have the position-dependent effect on the atom-field coupling. When \( \theta(\vec{r}_0) = 0 \) (or \( \frac{\pi}{2} \)), our two-band model becomes the one-band model.

### III. Spontaneous Emission Spectrum in the Free Space Vacuum

Our studied atom inside the PBG is a three-level \( \Lambda \)-type atom as shown in Fig. 1. The atom is assumed to be initially excited in state \( |2\rangle \). The transition between \( |2\rangle \leftrightarrow |1\rangle \) is considered to be near resonant with the band edge of the PBG reservoir, while the transition between \( |2\rangle \leftrightarrow |0\rangle \) is assumed to be far away from the PBG edge, and is coupling with the free space reservoir. We shall study the spontaneous emission spectrum of this latter transition.

In a rotating-wave approximation, the interaction Hamiltonian in the electronic dipole approximation \((\vec{k} \cdot \vec{r} \approx 0)\) can be written as

\[
H = \hbar \sum_\lambda g_\lambda e^{-i(\omega_\lambda - \omega_{20})t} |2\rangle \langle 0| a_\lambda + \hbar \sum_\kappa g_\kappa e^{-i(\omega_\kappa - \omega_{21})t} |2\rangle \langle 1| a_\kappa + \hbar \sum_\sigma g_\sigma e^{-i(\omega_\sigma - \omega_{21})t} |2\rangle \langle 1| a_\sigma + H.c. \tag{4}
\]

where \( g_\lambda = \frac{\omega_\lambda}{\hbar} \left( \frac{\hbar}{2\epsilon_0 \omega_\kappa V} \right)^{1/2} \frac{\vec{e}_{k\lambda}}{d_{21}} \cdot \vec{r}_{21} \) characterizes the coupling of the atom with the free-space vacuum modes \( \lambda \). Here \( V \) is the quantized volume of the field and \( \vec{e}_{k\lambda} \) is the polarization of the plane wave. Also, \( g_\kappa \) and \( g_\sigma \) determine the coupling strength of the atom with the air-band-reservoir modes \( \kappa \) and dielectric-band-reservoir modes \( \sigma \), respectively,

\[
g_{\kappa(\sigma)} = \frac{\omega_{21}}{\hbar} d_{21} \left( \frac{\hbar}{2\epsilon_0 \omega_{\kappa(\sigma)} V} \right)^{1/2} \vec{E}_{a(\sigma)}^* (\vec{r}_0) \tag{5}
\]

where \( \vec{E}_{a(\sigma)}^* (\vec{r}_0) \) are defined in Eq. (3). We also fixed polarization orientations of atomic dipole moments \( \vec{d}_{20} = d_{20} \hat{d}_{20} \) and \( \vec{d}_{21} = d_{21} \hat{d}_{21} \) [15]. For simplification, we assume all the coupling constants are real. \( \omega_{ij} \) indicates the atomic transition energy while \( \omega_\lambda, \omega_\kappa, \) and \( \omega_\sigma \) are reservoir mode energies.

In the single photon sector, the wave function of the system could be described as

\[
|\Psi(t)\rangle = \alpha_2(t) |2, \{0_\lambda, 0_\kappa, 0_\sigma\} \rangle + \sum_\kappa \alpha_{1\kappa}(\vec{r}_0, t) |1, \{0_\lambda, 1_\kappa, 0_\sigma\} \rangle + \sum_\sigma \alpha_{1\sigma}(\vec{r}_0, t) |1, \{0_\lambda, 0_\kappa, 1_\sigma\} \rangle + \sum_\lambda \alpha_{0\lambda}(t) |0, \{1_\lambda, 0_\kappa, 0_\sigma\} \rangle, \tag{6}
\]

where the state vectors \( |1, \{0_\lambda, 1_\kappa, 0_\sigma\} \rangle \) and \( |1, \{0_\lambda, 0_\kappa, 1_\sigma\} \rangle \) describe the atom in the intermediate state \( |1\rangle \) with a single photon in the modes of \( \kappa \) and \( \sigma \), respectively. Due to the
position dependence of the atom-field coupling inside the PBG, the probability amplitudes of the system in the states \(|1, \{0_\lambda, 1_\sigma\}\) and \(|1, \{0_\lambda, 0_\sigma\}\) are also functions of the relative position \(\vec{r}_0\) of the atom in a Wigner-Seitz cell of the PCs, and are assumed as

\[
\alpha_{1\kappa}(\vec{r}_0, t) = \alpha_{1\kappa}(t) \cos \theta(\vec{r}_0), \quad \alpha_{1\sigma}(\vec{r}_0, t) = \alpha_{1\kappa}(t) \sin \theta(\vec{r}_0).
\]

(7)

This assumption makes the probability of the system with atomic level \(|1\rangle\) to be \(|\alpha_{1\kappa}(t)|^2\).

Substituting Eqs. (4) and (6) into the the time-dependent Schrödinger equation, we obtain the time evolution of the probability amplitudes

\[
i\frac{d}{dt}\alpha_2(t) = \sum_\lambda g_\lambda \alpha_0\alpha(t)e^{-i(\omega_\lambda - \omega_0)t} + \sum_k g_k \alpha_{1\kappa}(t)e^{-i(\omega_\kappa - \omega_21)t}\cos^2 \theta
\]

\[
+ \sum_k g_k \alpha_{1\kappa}(t)e^{-i(\omega_\kappa - \omega_21)t}\sin^2 \theta,
\]

(8)

\[
i\frac{d}{dt}\alpha_0(t) = g_\lambda \alpha_2(t)e^{i(\omega_\lambda - \omega_0)t},
\]

(9)

\[
i\frac{d}{dt}\alpha_{1\kappa}(t) = g_k \alpha_2(t)e^{i(\omega_\kappa - \omega_21)t}\cos^2 \theta + g_k \alpha_2(t)e^{i(\omega_\kappa - \omega_21)t}\sin^2 \theta,
\]

(10)

where \(g_k = \frac{\omega_\kappa}{\hbar} d_{21} \left( \frac{\hbar}{2\omega_0^2 V} \right)^{1/2} \hat{d}_{21} \cdot \vec{E}_k\) is assumed to be a real constant.

With some algebra and defining the memory functions (or kernels) of the PBG reservoirs, we obtain the integrodifferential equation

\[
\frac{d}{dt}\alpha_2(t) = -\frac{1}{2} \alpha_2(t) - \int_0^t d\tau \alpha_2(\tau)K_1(t - \tau)\sin^4 \theta - \int_0^t d\tau \alpha_2(\tau)K_1(t - \tau)e^{-iD_\omega t}\sin^2 \theta \cos^2 \theta
\]

\[
- \int_0^t d\tau \alpha_2(\tau)K_2(t - \tau)\cos^4 \theta - \int_0^t d\tau \alpha_2(\tau)K_2(t - \tau)e^{iD_\omega t}\sin^2 \theta \cos^2 \theta
\]

(11)

where \(D_\omega = \omega_\kappa - \omega_\sigma\) is the air-band and dielectric-band frequency difference at a fixed wave vector. Because the most PBG DOS is contributed from the states near band edges, we can approximate \(D_\omega\) as the band-gap width \(\Delta\), \(D_\omega \cong \Delta = \omega_c - \omega_v\), where \(\omega_c\) (\(\omega_v\)) is the air-band (dielectric-band) edge frequency. Here the memory kernels are

\[
K_1(t) = \sum_k g_k^2 e^{-i(\omega_\kappa - \omega_21)t} = \beta^{3/2} \int d\omega \rho_\sigma(\omega)e^{-i(\omega - \omega_21)t},
\]

\[
K_2(t) = \sum_k g_k^2 e^{-i(\omega_\kappa - \omega_21)t} = \beta^{3/2} \int d\omega \rho_\alpha(\omega)e^{-i(\omega - \omega_21)t}
\]

(12)

where \(\beta\) is the atom-reservoirs coupling constant and density of states (DOS) \(\rho(\omega)\) is determined by the PBG model. When the case of a two-band isotropic model (Fig. 1 (b)) of
the photonic crystal is considered, the defined kernels could be determined and so be their Laplace transforms:

\[ \tilde{K}_1(s) \equiv \mathcal{L}[K_1(t)] = \frac{\beta_3^{3/2}}{2} \frac{e^{i\pi/4}}{\sqrt{s + i\delta_v}}, \quad \tilde{K}_2(s) \equiv \mathcal{L}[K_2(t)] = \frac{\beta_3^{3/2}}{2} \frac{e^{-i\pi/4}}{\sqrt{s + i\delta_c}} \]

with threshold detuning frequencies \( \delta_v = \omega_v - \omega_{21} \) and \( \delta_c = \omega_c - \omega_{21} \). The long time spontaneous emission spectrum of the transition \( |2\rangle \leftrightarrow |0\rangle \) could be given by \( S(\delta_\lambda) \propto |\alpha_0(\lambda)|^2 \) with the detuning frequency \( \delta_\lambda = \omega_\lambda - \omega_{20} \). After performing Laplace transform in Eq. (11) and the final-value theorem, we obtain

\[ S(\delta_\lambda) \propto \gamma |\tilde{\alpha}_2(s = -i\delta_\lambda)|^2, \quad \text{(13)} \]

where \( \tilde{\alpha}_2(s) \) is the Laplace transform of the probability amplitude \( \alpha_2(t) \) and is given by

\[ \tilde{\alpha}_2(s) = \left\{ 1 - \sin^2 \theta \cos^2 \theta \left[ \tilde{\alpha}_2(s + iD_\omega)\tilde{K}_1(s + iD_\omega) + \tilde{\alpha}_2(s - iD_\omega)\tilde{K}_2(s - iD_\omega) \right] \right\} \]

\[ \times \left[ s + \gamma/2 + \cos^4 \theta \tilde{K}_2(s) + \sin^4 \theta \tilde{K}_1(s) \right]^{-1} \quad \text{(14)} \]

with \( \tilde{K}_1(s) \) and \( \tilde{K}_2(s) \) being the Laplace transforms of the kernels expressed in Eq. (13). From the structure of Eq. (15), the emission spectrum depends on the modulus square of the sum of two amplitudes, and the detailed spectral shape of the radiated fluorescence is intimately controlled by quantum interference effects [8]. Besides, the kernels of Eq. (13) have singularities locating at \( s = -i\delta_c \) and \( -i\delta_v \), and the SE spectrum exhibits two dark lines at these singular points [3]. In the case of a free space vacuum no dark lines exist.

We show the spectrum as a function of detuning frequency \( \delta_\lambda \) for several gap (\( \Delta \)) widths in Fig. 2. A symmetric case where the atomic transition \( \omega_{21} \) is placed in the middle of the gap is chosen. There are three peaks in the spectrum, one main peak in the middle of the two side lobes for the symmetric case. The main peak is the atomic emission spectrum, which is referred as the free-space light and has a Lorentzian shape, in the free-space vacuum. Two side lobes are the atomic free-space transition coupling to the other transition within the PBG, which is referred as the PBG light. For the large (small) PBG, this coupling effect is negligible (strong). A free-space and Lorentzian-shaped spectrum centered at the resonant condition \( \delta_\lambda = \omega_\lambda - \omega_{20} = 0 \) is remained for the larger PBG [3]. If this coupling is stronger, the free-space light could emit through the channel of the PBG light strongly. Therefore, as the PBG is decreasing and this coupling becomes stronger, the radiated power of the main peak is lowering. While the widths and radiated powers of two side lobes are increasing (see
Fig. 2). There is no such intensity change of the free-space light in the previous model [3] of two independent PBG reservoirs. Note that, as the PBG becomes smaller, the field modes from the air band could interfere via the atom strongly with the modes from the dielectric band. Such a quantum interference is shown by two shallow kinks of the side lobes for $\Delta = 1$ in Fig. 2.

How the spectrum is affected by the relative position of an atom in a Wigner-Seitz cell is shown in Fig. 3. An asymmetric case for $\omega_{21}$ at the air-band edge is considered. The spectrum shape, being like the symmetric case, changes with the position-dependent parameter $\theta(\mathbf{r}_0)$. For increasing values of the $\theta$ parameter, where the atom is embedded at a position that the dielectric-band fields become stronger than the air-band fields, the free-space light makes a blue shift and its height decreases. Meanwhile, the radiated power of the PBG light from the air band increases and eventually becomes stronger than the free-space light. These changes of shape occur with not much change in the height of the side lobe from the dielectric band. To understand these spectrum changes, note that the free-space light is coupling with and could emit light through the PBG light strongly for a narrow gap. We have studied the case that the atomic transition is lying inside a narrow gap and close to the air-band edge. The coupling strength of the free-space light with the PBG light from the dielectric band is stronger than the coupling with the PBG light from the air band for the large $\theta$ parameter. This stronger coupling with the PBG light from the dielectric band pushes the free-space light toward the air band, and the free-space light could interfere with and emit through the PBG light from the air band strongly. This quantum interference effect is an intrinsic phenomenon of the PBG reservoirs. From the SE spectrum changing with the position of the embedded atom, we can realize the physical properties of the PBG-vacuum fields within the Wigner-Seitz cell and how the relative position of the embedded atom affects the coupling strength between the atom and the PBG reservoirs.

IV. CONCLUSIONS

We have studied the SE spectra of a three-level atom with $\Lambda$-configuration in PCs, even though these have been studied before. The previous double-band studies took the PBG reservoirs as two independent and incoherent reservoirs, and assumed equal strength of coupling between the atom and each individual reservoir of the PBG. With the consideration
of the phase difference between the air-band and dielectric-band fields, two band reservoirs are coherent and become a single coherent reservoir. The coupling strength of the atom with this coherent reservoir depends on the embedded position of the atom. The coherence phenomenon of the PBG reservoirs is shown in the SE spectrum, where the spectrum of the PBG light has shown quantum interference between the PBG reservoirs for the small PBG case. Such quantum-interference effect is from the multiple scattering of the fields inside the PCs. We believe that the PBG reservoirs is a single coherent reservoir and there is an intrinsic phenomenon of quantum interference between the PBG reservoirs in the SE spectrum.

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FIG. 1: (a) A Λ-type atom with transition frequency $\omega_{21}$ near resonant with a modified reservoir and $\omega_{20}$ occurring in free space. (b) Dispersion relation near the PBG edge. (c) DOS of the isotropic two-band model.

FIG. 2: The SE spectra for the symmetric cases. The parameters are $\gamma = 1$, $\theta(\vec{r}_0) = \pi/4$, $\delta_c = -\delta_v = 3$, and $\Delta = 6$ (dotted line); $\delta_c = -\delta_v = 1$, and $\Delta = 2$ (dashed line); $\delta_c = -\delta_v = 0.5$, and $\Delta = 1$ (solid line).


FIG. 3: The SE spectra for the asymmetric cases. The parameters are $\gamma = 1$, $\delta_c = 0$, $\delta_v = -1$, $\Delta = 1$, and $\theta(\vec{r}_0) = \pi/6$ (dotted line); $\theta(\vec{r}_0) = \pi/4$ (dashed line); $\theta(\vec{r}_0) = \pi/3$ (solid line).